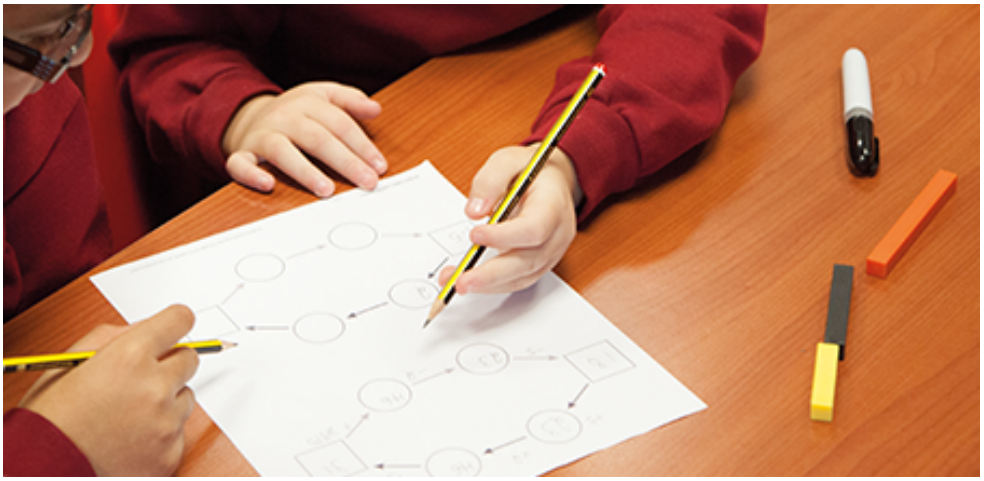
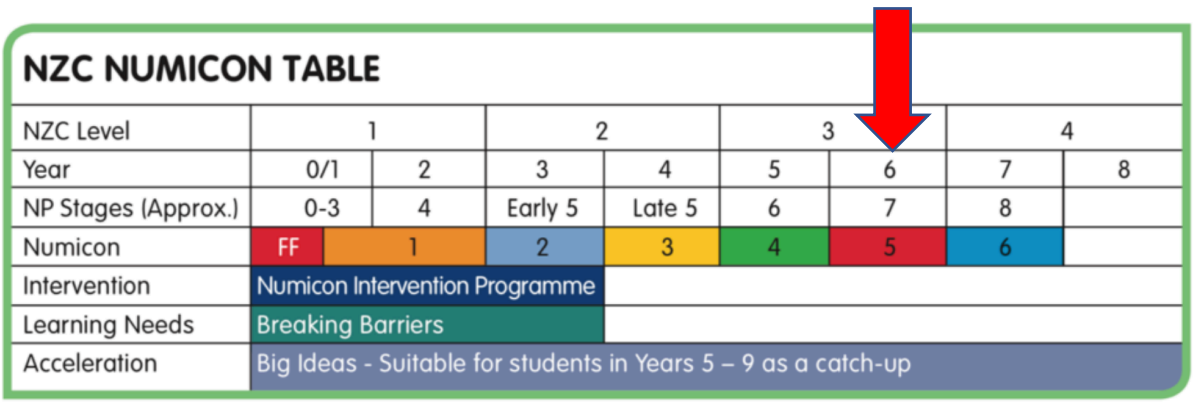
Numicon 5: Patterns and relationships - Using inverse relationships to solve problems



In this sample you will find:

- Information from the Implementation Guide on patterns
- A summary of Pattern and Algebra 2
- Opening discussion and problem-solving activity linking in previous experiences
- One of the activities
- An in-class and 'take home' activity to reinforce the teaching and build generalising skills
- An assessment you can use at the end of the week as a record of the children's learning. They have to apply what they have learned in these activities.



Implementation Guide

Pattern
An essential idea underlying all Numicon activities is that of pattern. Patterns are essential in mathematics for a very special reason: they enable us to imagine actions, events and sequences going on 'forever' without us having physically to work out and wait for each and every step.

It is patterns that allow us to generalize into the future. Counting is a good example. As we have invented a system for generating number names, we can imagine what it would be to count forever without ever actually having to do it. Most people know they could count to one million, without ever having done it. They know they could because they know the place value system; they know the patterns in number names that would enable them to go on forever were they ever called upon actually to do so.

Importantly, once children see the pattern that each next whole number is 'one more' than the previous one, they also know how counting things may go on forever – a vital generalization (called the 'successor relation') that allows children to work with collections of any size. As another example, by noticing the pattern that it doesn't matter which two numbers we choose to multiply first when we are multiplying three numbers together (a generalization we call the 'associative property'), children gain an important flexibility in calculating that allows them to find quick ways to multiply mentally. For example, 48 × 250 can become (4 × 12) × 250, which then becomes 12 × (4 × 250), which is then easy to calculate as 12 000. And because they have generalized in this way, they don't have to keep checking every example.

It is impossible to overestimate the importance of pattern to mathematical thinking. In fact, a very large part of algebra, often thought of as the most powerful branch of mathematics, consists of seeing, manipulating and generalizing from patterns. It is algebra that enables humans to launch space shuttles and bring them back successfully, by predicting and generalizing from patterns. It is important to remember that in all the key mathematical ideas discussed here, pattern and generalization are fundamental.

New Zealand's Mathematics Curriculum Opening statement:

Mathematics is the exploration and use of patterns and relationships in quantity, space and data

Pattern and Algebra 2: Using inverse relationships to solve problems

Key mathematical ideas Inverse, Adding, Subtracting, Multiplying, Dividing, Pattern, Mathematical thinking and reasoning

Educational context

The key theme of this activity group is that of inverse elationships. Doing mathematics is essentially about studying relationships, and so 'making connections' of every kind is always very important for children. The work here builds on the *Number, Pattern and Calculating 4 Teaching Resource Handbook*, Pattern and Algebra 2.

Inverses are essentially about 'doing and undoing' and so here children are invited to work on adding and subtracting and on multiplying and dividing as pairs of operations that 'undo' each other. Sometimes this will feel like 'working backwards' through a problem.

This lays the foundations for later work in which children will be introduced to pairs of inverse functions, e.g. converting from metric to imperial units and then back again (e.g. in the *Geometry, Measurement and Statistics Teaching Resource Handbook*, Measurement 1). Importantly, children will also discover that some actions are irreversible – they are impossible to undo exactly – and that it is important to know which actions these are.

Throughout, encourage children to use plenty of time, experimentation and discussion to think out how to 'work backwards'. It can feel very strange, but also like a good puzzle if the classroom atmosphere is right. Encourage children to use apparatus and imagery to support their reasoning as they work.

Learning opportunities

- To multiply and divide numbers mentally, drawing upon known facts.
- To solve missing number problems by using the inverse relationship between adding and subtracting.
- To solve missing number problems by using the inverse relationship between multiplying and dividing.
- To solve problems involving

Terms for children to use

inverse facts/relationships/calculations/operations, arithmagon, factor, multiple, common factors, working backwards, compare, adding, subtracting, multiplying, dividing, double, halve, solution, trial and improvement, column method, number trio

Assessment opportunities

Look and listen for children who can:

- Use the terms for children to use effectively.
- Use adding and subtracting and the inverse relationship between them flexibly and fluently to solve number puzzles and check solutions.
- Use multiplying and dividing and the inverse relationship between them flexibly and fluently to find solutions to number puzzles and check solutions.
- Recall number facts fluently and use the inverse relationships between adding and subtracting and between multiplying and dividing to complete calculations with missing numbers.
- Use recalled number facts flexibly to solve problems by working backwards.
- Identify inverse operations in number 'tricks' and explain that these will cancel each other out.
- Create number tricks using inverse operations.

NPC Milestone 3

- Use the inverse relationships between adding and subtracting, and multiplying and dividing, to complete calculations with missing numbers (NPC 5:3c)

Explorer Progress Book 5b, pp. 6–7

After completing work on this activity group, give small focus groups of children their Explorer Progress Books and ask them to work through the challenges on the pages. As children complete the pages, assess what progress they are making with the central ideas from the activity group. Refer to the assessment opportunities for assistance.

Explore More Copymaster 3: Missing Number Game!

After completing work on Activity 3, give children Explore More Copymaster 3: Missing Number Game! to take home.

Pupil Book 5, pp. 54–57

These pages in the Pupil Book provide further practice and challenging questions. You can use them to follow up the activities and deepen the learning.

Focus activities

1. Exploring arithmagons
2. Using inverse facts to complete arithmagons
3. Completing calculations with missing numbers
4. Finding missing digits in written calculations
5. Solving problems by working backwards
6. Using number loops
7. Think of a number

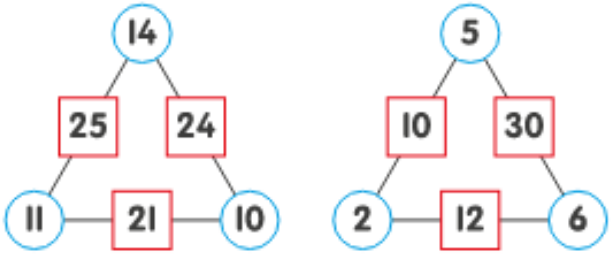
Opening Activity Step 1

Pattern and Algebra 2·1 & 2·2

Inverse relationships

Practice

1 Can you explain the relationships between the numbers in the circles and the numbers in the squares, in these arithmagons?



This is the opening question in the Numicon 5 Pupil book to promote discussion.
Allow children to work in pairs or small groups before reconvening as a class to discuss them further.

Step 2
Discuss the above then show how they develop their thinking further.
Talk with children about the column methods of adding and subtracting they revised in previous activity groups (e.g. Calculating 3 and Calculating 4). Show them some examples with missing digits (see image), telling them that all the carrying figures are also missing.

Help them to find the missing digits by working through the calculations one column at a time.

	2	9	□
+	4	□	5
	7	4	3

	□	7	6
+	2	3	□
	8	1	2

	5	1	3
−	2	□	□
	2	2	8

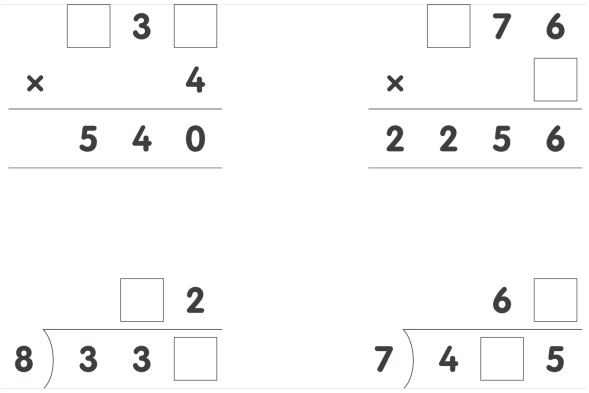
	□	0	4
−	4	2	□
	1	8	2

Step 3
Look and listen for children who can work logically through the problems and explain how to find each missing number.

Look and listen for children who suggest using inverse calculations to identify the missing digits, as well as to check answers, e.g. to check we can work out 298 + 445 = 743 they could do the subtracting calculation 743 – 298 = 445.

Step 4
Repeat with multiplying and dividing calculations (see image 1).

Work with children to find the missing digits. Encourage them to talk about how inverse facts, factors and multiples help us to solve each problem, e.g. in the first multiplying calculation they might find a multiple of 4 that ends in 0, like 20, and then divide 20 by 4 to find the missing ones digit, 5.



Paired or individual work

Have ready: calculations with missing digits (see image)

Provide calculations with missing information for children to complete, (see image).

Alternatively, ask children to set each other similar problems.

Now link to:
Numicon Pupil Book 5 pp. 54–57
Numicon Pupil Book 5 Answer Book p. 33
Explore More Missing Number Game

End the Maths time with one of these whole class discussions:

- Discuss with children how and when the mathematics they have been learning could help them in solving problems.
- Show children calculations and ask them to write their inverses, e.g. (5 × 4) + 7 = 27 is (27 − 7) ÷ 4 = 5.
- Challenge children to complete a list of empty box problems, e.g. 2056 + □ = 2085, 560 ÷ □ = 8, against the clock. Include examples where the empty box represents the calculation symbol, e.g. 1800 □ 30 = 60.
- Give children plenty of practice solving ‘I’m thinking of a number’ clues, e.g. ‘I’m thinking of a number. When you add 56 to it, the answer is 100.’
- Provide alternative instructions for children to make number loops, or ask children to come up with instructions of their own.
- Provide children with a list of calculations that need to be checked. Ask them to rewrite each one as an inverse calculation and look for errors.
- Encourage children to routinely check calculations with inverse facts.

Missing Number Game!

How this will help your child

- This activity will help your child to complete calculations with missing numbers.
- It will support them in understanding the relationship between adding and subtracting and between multiplying and dividing.

Words and phrases to use

add, sum, total, subtract, multiply, divide, difference, equals, more, less, increase by, decrease by

You will need

- 2 paper clips
- 2 pencils
- Scissors
- A counter or button for each player

During the activity, look at what your child can do

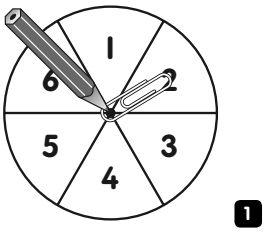
- Use number facts to work out the missing number in a calculation.
- Keep a running total.

What to do

- Cut out the cards from the Missing Number Game! sheet and give one to your child and keep one for yourself. Then each write a starting total of 100 at the top of the 'Running total' column.
- Place a counter for each player on the start position of the game board.
- Ask your child to spin the number spinner, using a paper clip and a pencil, and move their counter that many spaces along the game board, e.g. 2. **1**
- Then they write the calculation shown on the square of the game board on their card. Ask them to work out the missing number and write it on their card.
- Now, ask your child to use a paper clip and a pencil to spin the spinner with words on. They need to either add or subtract the missing number from their running total, e.g. if they land on □ + 59 = 100 and then spin 'increase by', they would add 41 to their running total, so as to increase their running total by 41. **2**
- Ask your child to record this on their card. **3**
- When it is your turn, ask your child to help you to calculate the missing number.
- When both players have completed five turns, the person with the highest running total is the winner!

Next steps …

- Play the game again with different starting totals. Have ten turns each.
- Make your own board, with different calculations.
- Ask your child kitchen problems with missing information, e.g. 'Two equal-sized glasses of water hold 200 ml, how much does one glass hold?'



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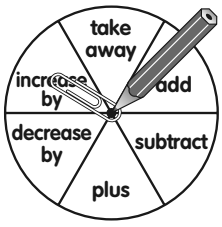


2

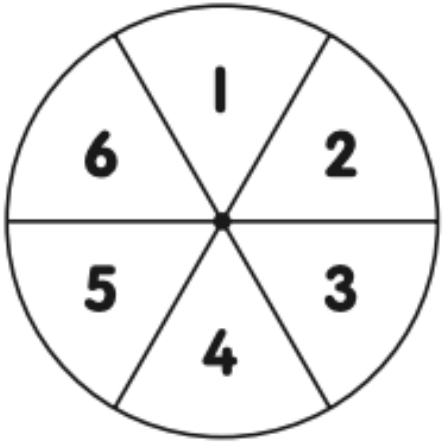
Calculation	Missing number	+ or –	Running total
			100
□ + 59 = 100	41	+	141
105 ÷ □ = 5	21	–	120

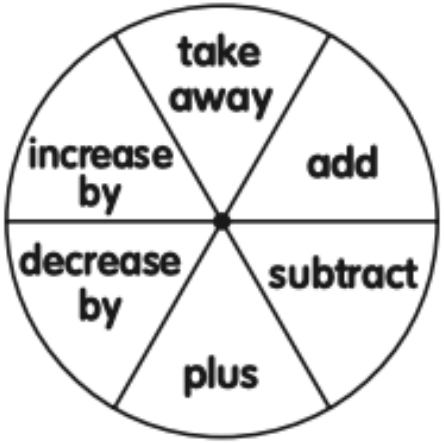
3

Name ..... Date ...../...../.....

Missing Number Game!

START →	2 × □ = 120	□ + 59 = 100	70 − □ = 20	□ + 45 = 105
20 + □ = 54				105 ÷ □ = 5
35 + □ = 90				33 + □ = 90
□ ÷ 3 = 20				2 × □ = 126
65 + □ = 105				65 + □ = 109
□ × 2 = 70				3 × □ = 93
90 − □ = 43	4 × □ = 120	88 ÷ □ = 8	111 − □ = 55	120 ÷ □ = 4





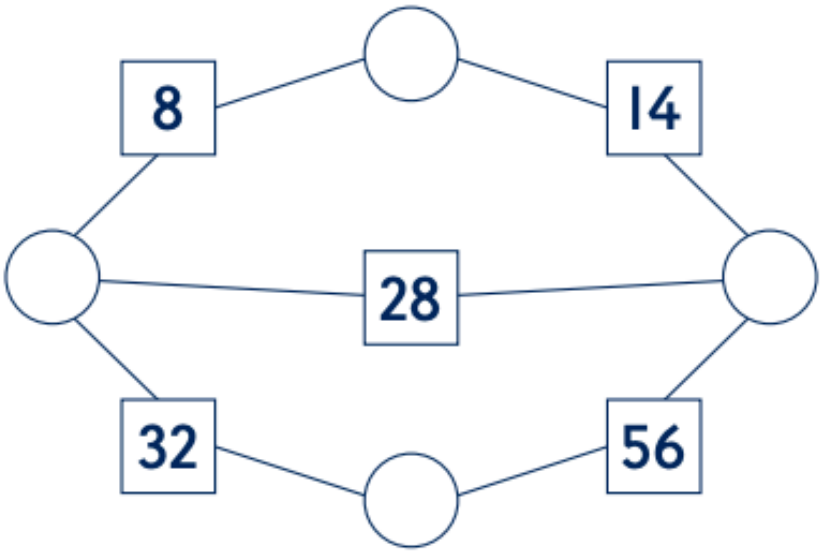
Calculation	Missing number	+ or −	Running total

Calculation	Missing number	+ or −	Running total

Number, Pattern and Calculating 5
Patter and Algebra 2, Using inverse relationships to solve problems

Date ..../..../.....

Number Puzzle



In this puzzle, numbers in the circles either side of a square were multiplied together to give the answer in the square.

Can you work out what the numbers in the circles must have been?

Teacher notes