numicon
a clear image for maths

Numicon 4: Fractions and recognizing part-whole relationships

In this sample you will find:

- Information from the Implementation Guide on Fractions. This may be a longer read than previous examples, but one which you will appreciate and use in your teaching giving insight on understanding fractions. You will also notice the *progression* through Numicon books 1-3 as prerequisites for learning at this level.
- A summary of Numbers and Number System 5
- Opening discussion and problem-solving activity linking in previous experiences of fractions
- One of the activities in NNS that you can also link in Time, Measurement and Geometry
- An in-class and 'take home' activity to reinforce the teaching and build generalising skills
- An assessment you can use at the end of the week as a record of the children's learning. They have to apply what they have learned in these activities.

NZC NUMICON TABLE

NZC Level	1		2		3		4	
Year	0/1	2	3	4	5	6	7	8
NP Stages (Approx.)	0-3	4	Early 5	Late 5	6	7	8	
Numicon	FF	1	2	3	4	5	6	
Intervention	Numicon Intervention Programme							
Learning Needs	Breaking Barriers							
Acceleration	Big Ideas - Suitable for students in Years 5 – 9 as a catch-up							

Implementation Guide

Fractions and recognizing part-whole relationships

Fractions are a complex idea, and there are several different symbolic ways of representing what are essentially the same kinds of numbers (decimals, ratios, percentages). One of the key challenges for teachers at this stage is to guide children to understanding that common fractions, decimal fractions, percentages and ratios are essentially just different forms of notation for the same 'rational' numbers, and that 'ratio' is at the heart of multiplicative thinking.

In Number, Pattern and Calculating 1 and 2 common (or 'vulgar') fractions and their notation began to be introduced, and were related to whole numbers through representation as distances along a number line. In Number, Pattern and Calculating 3, the terms 'numerator' and 'denominator' were formally introduced, counting on and back in fractions was further developed, and fractions (< 1) with the same denominator were added and subtracted. In Number, Pattern and Calculating 4, common fractions feature more prominently, and key associated developments involve the introduction of decimal fractions, mixed numbers and improper fractions. Importantly, recognizing the equivalence of a range of fractions (< 1) is also emphasized.

Typically for young children, fractions arise in measuring situations (which include those involving 'sharing'). Measuring is always approximate and for this reason we find ourselves quickly needing parts of whole units to describe amounts accurately. The moral imperative for 'fair' shares usually draws children easily to the view that fractions are (and indeed should be) equal parts of a whole.

The two main ways in which children have initially experienced fractions in the Numicon teaching programme are as 'descriptors' and as 'operators' – or as adjectives and verbs. As a consequence of certain dividing calculations (or measuring tasks), children met fractions as adjectives, for example as 'twenty-six-and-a-half somethings', or as the description of a distance, for example as 'halfway' between 26 and 27 on a measuring scale. On the other hand, the invitation to 'find half of twenty- six' was an invitation to halve 26 – the same word functioned as a verb, as an implied instruction to do something.

Importantly also, from Number, Pattern and Calculating 2 onwards, children began to meet fractions as numbers, signalled by the use of fraction words as nouns. This was another key step inviting children to move from processes to objects. From 'halving' situations in which there was action (the process of halving a pizza) and we used a verb, 'to halve', we moved to an adjective, 'half', to describe the result of the action ('a half pizza'), and finally we did a mysterious thing, which was to start talking about 'a half' as an abstract object, a noun naming a distance along a number line, from which context the pizza has, confusingly, disappeared altogether.

It cannot be emphasized too strongly that turning 'fractions of something' into just 'fractions' as mathematical objects is a seriously strange thing for children to join in with. The models and imagery we offer children to get used to these strange new objects are crucial and once again children need plenty of time, opportunity and imagery to get fully used to our new ways of talking and communicating about them.

In Numicon, children first meet fractions as descriptors in their answers to dividing calculations, and as operators in instructions to 'halve', 'find a sixth of' and so on in scaling situations. This in fact mirrors something that happened earlier with whole numbers: initially children met whole numbers in the process of counting; then as adjectives, for example 'six sweets'; and it was only later they had to understand the use of these words as abstract nouns, for example 'six', giving simply the name of a number idea.

Fortunately for teachers of young children, at the beginning it was only necessary to introduce the idea of a fraction as a proportion of something, where the 'something' could always be specified. Thus in scaling situations such as

'find a third of X' the 'X' was always clear, and in measuring (including sharing) situations what the fraction was of would be equally explicit.

However, for teachers of older children, in Number, Pattern and Calculating 4, extending work on 'parts of wholes' to fractions as objects, as distances filling spaces between whole numbers on the number line and using the terms 'half', 'two-thirds' and so on as nouns is much more complex. The difficulty for children of making these moves in their thinking and communicating should never be underestimated. Allow all the time it takes for them to question, discuss and try out for themselves these new ways of communicating.

Since the first moves to thinking of fractions as number objects were taken in Number, Pattern and Calculating 2, it was possible in Number, Pattern and Calculating 3 to take the first steps in adding and subtracting fractions (with the same denominator). In Number, Pattern and Calculating 4, we are able to take a next crucial step in recognizing equivalent fractions (< 1) – a significant consequence of the fact that fractions are essentially *ratios*. Just as the ratio 1:3 is equivalent to the 12 ratio of 2:6, so $_{3}$ is equivalent to $_{6}$.

Use of Numicon Shapes, number rods and objects arranged in arrays and visual imagery such as diagrams and number lines is essential to communicating about fractions, as is the use of everyday and realistic contexts that children can relate to. Measuring scales are particularly useful.

When using the terms 'numerator' and 'denominator' with older children it can be helpful to explain their sense. A de*nom*inator gives a common fraction its name – it tells you what kind of a fraction it is. A *num*erator tells you how many of this kind of fraction you have. There is always a history to how we do things in mathematics.

Equivalence

Equivalence is one of the most important mathematical ideas of all and yet it is often the case that not enough explicit attention is paid to it. Children actually work with equivalence from very early on in their thinking. Equivalence is also the big idea behind the Piagetian conservation judgements children make – two collections of (say) eight objects are numerically equivalent to each other, however different they look. In fact, it is difficult to see how children could have made any progress in their learning of language (long before school) without noting various equivalences in the sounds and expressions they heard and made.

Equivalence literally means 'equal value'. We might add that the most interesting and important instances of equivalence occur when two or more things are of equal value, but look different. In early calculating, there are three important occasions when children face explicit instances of equivalence: when they encounter quantity value and column value, the introduction of the '=' sign 123 (which means 'is equivalent to') and when they meet fractions (= =). However, there are numerous other occasions when they need to see equivalence when teachers are usually less explicit.

For their mental arithmetic strategies to make sense, children have to be able to see what are called the 'decompositions' of any number as equivalent to each other, for example 9 = 1 + 8 = 3 + 6 = 10 − 1. In measuring, equivalences between units (100 cm = 1 m, 1" = 2·54 cm) are at the heart of being able to understand systems of measurement, as well as the idea of using any kind of repeated unit, for instance all 'metres' are as good as each other.

When young children seem not to understand something that is clear to us in arithmetic, there is often an equivalence we see that they do not, or vice versa. Don't forget that equivalence is about things that are worth the same, but which look different; very often we find it difficult to see past appearances. We tell children that 'tens' are not 'ones' even though their numerals look the same, and yet we also tell them that tens and ones are 'the same thing' (equivalent) when it comes to breaking up a ten to make ten ones.

There is also a language problem here. As we find it difficult to use the word 'equivalence' with young children, we tend to resort to an easier word, 'same', when talking about equivalence; we say, 'It's the same thing.' Unfortunately, equivalence does not quite mean 'the same thing' – it means equal value, different appearance.

In Number, Pattern and Calculating 4, another important equivalence is introduced – equivalence between common fractions and decimal fractions. It is important to children's developing understanding that they realize '0·5' and '$\frac{1}{2}$' are different ways of referring to the same (rational) number. In their later school years, children will also understand that '50%' is equivalent to both 0·5 and $\frac{1}{2}$.

Thinking mathematically

Numicon is aimed at developing children's mathematical thinking. To do this, we work on children's mathematical communicating. That is because thinking and communicating are inextricably woven together. Children's mathematical thinking develops as they learn to join in with the ways in which we, as expert mathematical thinkers, communicate.

So, in order to appreciate what we are trying to do with Numicon, it is important to understand our view on mathematical thinking; without such an understanding it will be difficult to use the activities effectively with children.

This section sets out what we consider to be the essence of mathematical thinking.

Every mathematics lesson children experience involves them working on and with patterns of relationships, that is, working on the structures in situations. For instance, there is an important structure in the way that we name numbers – we call this 'place value', a structure in which we can shift numerals along one position to the left in order to multiply a value by 10. At other times, we might want children to work on the structure of relationships between numbers – such as factor relationships – where the place value properties of number names are almost entirely irrelevant and we might only be interested in, for example, even and odd properties.

The best mathematics teaching occurs when the teacher's and children's shared focus on the particular structures of a situation renders that situation more predictable for children. It pays for teachers to be clear in every lesson about the particular structures – that is, the relationships – children need to attend to.

Effective teachers of mathematics have always recognized that learning mathematics is about more than learning facts and calculation techniques; it is also about learning how mathematics is actually practised, about mathematics as activity.

So, although children will learn about number relationships and calculations, crucially they also need to learn how to go about doing mathematics in a variety of real-world situations.

New Zealand's Mathematics Curriculum Opening statement:

Mathematics is the exploration and use of patterns and relationships in quantity, space and data

Numbers and the Number System 5: Fractions and recognizing part-whole relationships

Key mathematical ideas Fractions, Equivalence, Mathematical thinking and reasoning

Educational context

The activities in this group consolidate and build on children's earlier work with fractions in the *Number, Pattern and Calculating 3 Teaching Resource Handbook*. A wide variety of images and measuring contexts are used to provide a foundation from which children can gradually generalize about fractions as numbers and as distances along a number line. The activities encourage children to notice the relationship between the numerator and denominator to help them compare and order fractions. Children also develop previous work with calculating and prepare for connecting common fractions with their equivalent expressions in decimal fraction notation. Importantly, children begin to add and subtract fractions with a common denominator, initially within realistic contexts.

Learning opportunities

- To explore fractions in different contexts.
- To generalize about a half, a quarter and three quarters.
- To develop understanding of fractions equivalent to a half.
- To know that, when comparing fractions with a common denominator, the larger numerator represents the larger fraction.
- To understand, when comparing unit fractions, that the larger the denominator, the smaller the parts.
- To add and subtract fractions with the same denominators.

Terms for children to use

fraction, half, set, part, whole, equal parts, one out of two equal parts, numerator, denominator, quarter, three quarters, array, equivalent fraction, equivalent to, relationship, out of, thirds, fifths, sixths, sevenths, mixed number

Assessment opportunities

Look and listen for children who can:

- Use the terms for children to use effectively.
- Recognize and name halves (and quarters) as equal parts of any whole.
- Explain patterns seen in relationships between fractions equivalent to a half.
- Explain that, the larger the denominator, the smaller the part.
- Illustrate and solve adding and subtracting calculations involving fractions.

NPC Milestone 3

- To know that, when comparing fractions with a common denominator, the larger numerator represents the larger fraction (NPC 4:3c)
- To make connections between fractions of a shape or fractions of one whole and fractions of a length or of a set of objects (NPC 4:3d)

Explorer Progress Book 4b, pp. 10–11

After completing work on this activity group, give small focus groups of children their Explorer Progress Books and ask them to work through the challenges on the pages. As children complete the pages, assess what progress they are making with the central ideas from the activity group.
Refer to the assessment opportunities for assistance.

Explore More Copymaster 13: Fraction Finder

After completing work on Activity 2, give children Explore More Copymaster 13: Fraction Finder to take home.

Pupil Book 4, pp. 62–65

These pages in the Pupil Book provide further practice and challenging questions. You can use them to follow up the activities and deepen the learning.

Focus activities

1. Generalizing about halves and quarters
2. Fractions equivalent to a half
3. Comparing fractions with different numerators and the same denominators
4. Comparing unit fractions with different denominators
5. Adding and subtracting halves and quarters
6. Adding and subtracting fractions beyond 1

set	equivalent	fourths	thirds
denominator	array	halves	fifths
numerator	part	relationship	sixths
equal parts	three	equivalent to	sevenths
whole	quarter	fraction	mixed number

Tips:

- Create flashcards from the list of words and terms from the Summary page to display on your Maths wall.
- Add photos to illustrate them.
- Use the words in Literacy activities.

Ask children to talk about sharing the prize within a team. Look and listen for those who realize the effect that having a team with more members would have on the proportion of the prize each player would get, and who can use fractions in their conversation.

Step 2
Role play with children building teams from
1 to 12 players in size. For each team in turn, discuss what proportion of the prize each player would get. Ask children to write this down as a fraction, and to talk about the numerator and the denominator. Then ask them to explain what happens to the amount of the prize each player gets as the teams
get bigger.

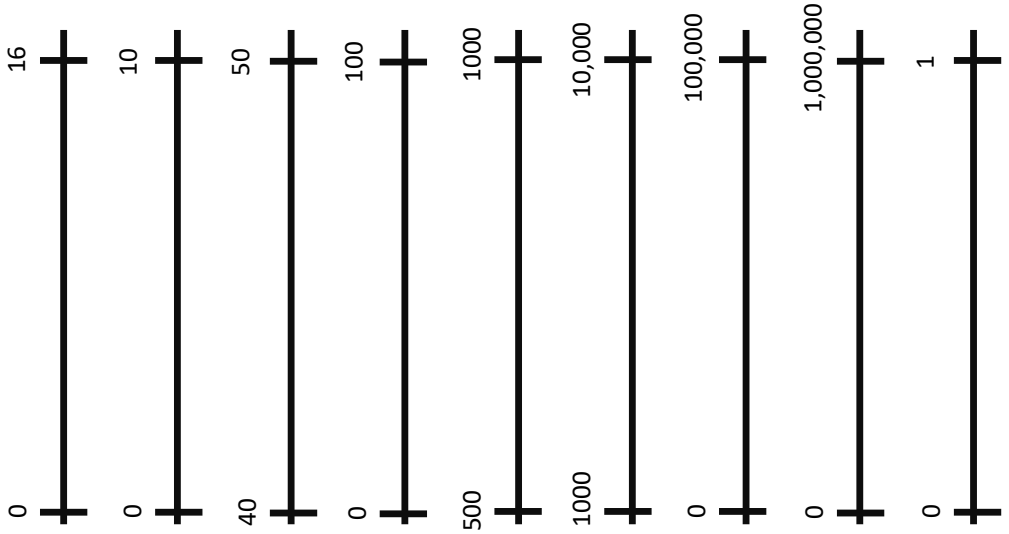
Give children a chance to explore this scenario further with different-coloured strips of paper representing the prize. First, ask one child to take the first strip of paper, representing the whole prize, and display it for children to see. This could be labelled ‘one whole’ or ‘1’. Agree this is the amount that one player would win if they played on their own (and won).

Step 3
Select another child to join the first, making a team of two, and give them another strip of paper the same length. Agree that this can be folded in half. Ask them to cut or mark halfway. Label each piece with the word ‘half’ and the fraction ‘1/2, then display it under the first strip.

Steps 4 - 7

Increase the activity to fourths, eighths and thirds then compare them.

1. Mark the halfway point and write in the number.
2. Now do the same for the quarter and three-quarter marks

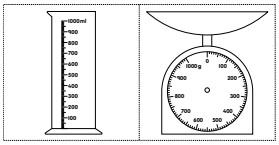


A1 Measuring scales

Half way

Three-quarter way

Quarter way



Talk to each other to decide where the arrows should be placed between 0 and 1000.

Put the arrow cards to the halfway mark, quarter way mark and three quarters way mark.

What do you notice about the half way and quarter points of these measuring scales?

Now draw these in your books and label the marks where you put the arrows.

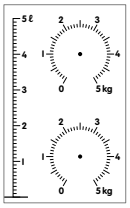
Now put the arrows to show halfway between 0 and 500, 300, 200, …

A2 Measuring scales

Half way

Three-quarter way

Quarter way



Talk to each other to decide where the arrows should be placed

Put the arrow cards to the halfway mark, quarter way mark and three quarters way mark.

What do you notice about the half way and quarter points of these measuring scales?

Now glue these in your books and label the marks where you put the arrows.

B. Halves up to 20

Shuffle the cards. Lay them out.

Take turns make a fractions to 20 showing the relationship of HALF.

As you make them, write them in your book. Continue the pattern until 100!

1

2

B. Halves up to 20

Shuffle the cards. Lay them out.

Take turns make a fractions to 20 showing the relationship of HALF.

As you make them, write them in your book. Continue the pattern until 100!

1

2

Fraction Finder

How this will help your child

- This activity will give your child practice in finding fractions that are equivalent to a half.
- It will help them to say fraction names.

Words and phrases to use

fraction, a half is one out of two equal parts, equivalent to a half, quarter, sixth

You will need

- Scissors
- Counters or buttons

During the activity, look at what your child can do

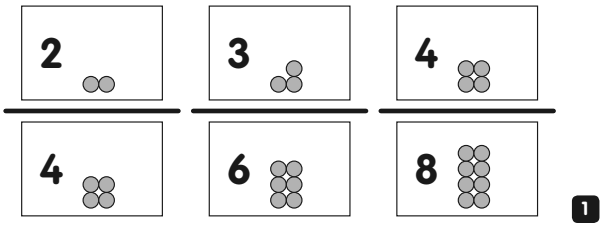
- Recognize fractions equivalent to a half.
- Say the names of these fractions

What to do

- Look at the $\frac{1}{2}$ fraction at the top of the Fraction Finder sheet. Describe a half by saying 'one out of two equal parts'.
- Cut out the number cards.
- Ask your child to use the cards to make fractions that are equivalent to a half ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{7}{14}$, $\frac{8}{16}$, $\frac{9}{18}$, $\frac{10}{20}$). **1**
- Encourage them to look carefully at the Numicon Shape patterns made of circles on each card.
- Shuffle the cards and deal ten each.
- Choose a card to put on the top or bottom of the empty fraction on the sheet. **2**
- Ask your child to find another card to make the fraction equivalent to a half.
- If they can, they say the fraction out loud, e.g. 'five tenths' and take a counter.
- If they can't, they replace the card on the top or bottom with one of their own.
- Take it in turns to put down cards to make fractions equivalent to a half.
- If the cards run out, shuffle and deal again.
- The winner is the first to get five counters.

Next steps...

- Make each number 10 times bigger by adding a 0 to the end of each number, e.g. 1 becomes 10 and 20 becomes 200. Then play again.
- Write half the match time for sports played in halves as a fraction of the whole match time, e.g. football $\frac{45}{90}$.



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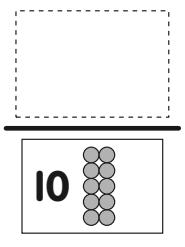
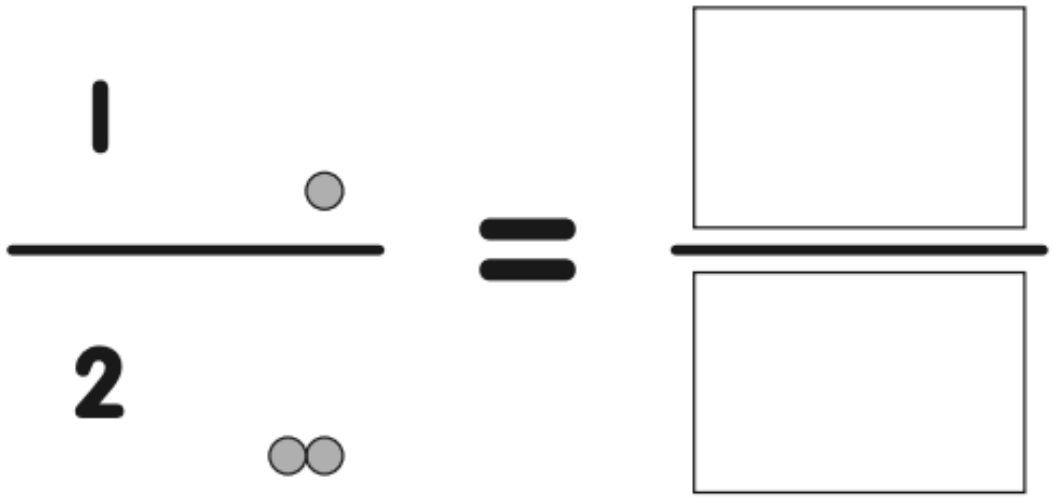


2

Step 1 Opening activity:

Fraction Finder



1	2	2	3
4	4	5	6
6	7	8	8
9	10	10	12
14	16	18	20

Present children with a sharing scenario, e.g. a competition where the prize is a mystery pot of money and you can enter on your own or as one of a team of, say, 2, 4 or 8 players.

Have ready: apparatus for representing fractions

Date ...../...../.....

Representing Fractions

Can you show four different ways of representing this fraction?

$$\frac{3}{4}$$

You might like to use apparatus, number lines, groups of objects or equivalent fractions.

Teacher notes

Date ...../...../.....

Half-time

Sarah has to buy the oranges for half-time at the school netball match. Each child in the two teams gets a quarter of an orange.

If there are 7 players in each team, how many oranges does Sarah need to buy?

Can you explain how you worked out your answer?

Teacher notes